Reproducing Kernel Hilbert Spaces
Syllabus

General information

Location: Cupples I Room 111
Time: MWF 10-11am
Professor: Greg Knese
Office location: Cupples I room 214
Office hours: to be announced
Email: geknese at wustl dot edu

Course description

Reproducing kernel Hilbert spaces have developed into an important tool in many areas, especially statistics and machine learning, and they play a valuable role in complex analysis, probability, group representation theory, and the theory of integral operators. This course will offer a unified overview providing detailed examples of applications, as well as covering the fundamental underlying theory, including chapters on interpolation and approximation, Cholesky and Schur operations on kernels, and vector-valued spaces. Prerequisites: 4111-4112. Suggested: 5051-52.

Textbook

An Introduction to the Theory of Reproducing Kernel Hilbert Spaces. Vern Paulsen and Mrinal Raghupathi. 2016, Cambridge.

An early version of this book can be found here:
https://www.math.uh.edu/~vern/rkhs.pdf

We should be able to finish this book and more on to other topics, such as those found in Agler and McCarthy's book "Pick interpolation and Hilbert function Spaces". In particular, we may look at complete Pick spaces and interpolation.

Grade breakdown

Attendance: 25%
Participation: 25%
Presentation: 50%

Presentations

Everyone in the course will be required to give a presentation on a topic or paper that uses RKHS or positivity in an essential way. Presentations will be given at the end of the course April 5-26. Please send me your presentation topic and date by March 8. Topics/papers as well as presentation dates will be first come-first serve. Here is a list of papers/topics that could make good presentations:

1) Aronszajn, N.; Smith, K. T.
Characterization of positive reproducing kernels. Applications to Green's functions.
Amer. J. Math. **79** (1957), 611–622.

2) Bell, Steve; Ligocka, Ewa
A simplification and extension of Fefferman's theorem on biholomorphic mappings.
Invent. Math. **57** (1980), no. 3, 283–289.
(Note this paper has several follow-ups and improvements.)

3) Quillen's theorem
a) Catlin, David W.(1-PURD); D'Angelo, John P.(1-IL)
A stabilization theorem for Hermitian forms and applications to holomorphic mappings.
Math. Res. Lett. 3 (1996), no. 2, 149–166.

b) Quillen, Daniel G.
On the representation of hermitian forms as sums of squares.
Invent. Math. **5** 1968 237–242.

c) Drouot, Alexis(1-CA); Zworski, Maciej(1-CA)
A quantitative version of Catlin-D'Angelo–Quillen theorem. (English summary)
Anal. Math. Phys. 3 (2013), no. 1, 1–19.

4)
Belton, Alexander(4-LANC-NDM); Guillot, Dominique(1-DE-NDM); Khare, Apoorva(1-STF-NDM); Putinar, Mihai(1-UCSB-NDM)
Matrix positivity preservers in fixed dimension. (English, French summary)
C. R. Math. Acad. Sci. Paris 354 (2016), no. 2, 143–148.

Belton, Alexander(4-LANC-NDM); Guillot, Dominique(1-DE-NDM); Khare, Apoorva(1-STF-NDM); Putinar, Mihai(4-NWCT-NDM)
Matrix positivity preservers in fixed dimension. I. (English summary)
Adv. Math. 298 (2016), 325–368.

5) Matrix monotone functions
Löwner, Karl
Über monotone Matrixfunktionen. **(German)**
Math. Z. 38 (1934), no. 1, 177–216.

Donoghue, William F., Jr.
Monotone matrix functions and analytic continuation.
Die Grundlehren der mathematischen Wissenschaften, Band 207. *Springer-Verlag, New York-Heidelberg,* 1974. v+182 pp.

6) Batayneh, Fawwaz(1-CLEM); Mitkovski, Mishko(1-CLEM)
Localized frames and compactness. (English summary)
J. Fourier Anal. Appl. 22 (2016), no. 3, 568–590.

7) RKHS and wavelets or harmonic analysis

8) Model spaces:
https://arxiv.org/abs/1312.5018

9) RKHS and stochastic processes

10) de Branges spaces of entire functions

11) Interpolating sequences in spaces with the complete Pick property
Alexandru Aleman, Michael Hartz, John E. McCarthy, Stefan Richter

https://arxiv.org/abs/1701.04885

12) Many papers available on Pick interpolation and variations