Math 470-Fall 2023

Analytic Combinatorics

$$\frac{1}{n!} = \frac{1}{2\pi i} \int_{|z|=n} e^z \frac{dz}{z^{n+1}} \sim \left(\frac{e}{n}\right)^n \frac{1}{\sqrt{2\pi n}}$$

General information

Class time: MWF 4-4:50pm

Class location: Cupples II room 230

Instructor: Greg Knese

Office hours: MW 12-1pm, Th 11am-12pm

Email: geknese at wustl dot edu

Course description

Analytic combinatorics is the study of counting sequences associated to combinatorial configurations. The course breaks up into two complementary components. First, we systematically encode and study counting problems through the use of power series (generating functions). Second, the analytic properties of these power series are used to understand the growth of counting sequences. Some examples are counting structured strings, functions, permutations, trees, and lattice paths. The principal analytic technique will be complex analysis and the course will include a user's self-contained introduction to complex analysis. Time permitting, we may study counting problems with parameters which naturally leads to multivariable generating functions and allows us to investigate statistical properties of counting sequences. Prerequisites: Math 310 or CSE 240.

Textbook

Analytic Combinatorics by Philippe Flajolet and Robert Sedgewick.

Available here: http://ac.cs.princeton.edu/home/AC.pdf

The book is so big (and interesting) that it may be worthwhile to get a physical copy.

See also the extensive website with videos

http://ac.cs.princeton.edu/online/

Course plan

Chapter I: Combinatorial structures and ordinary generating functions. Chapter II: Labelled structures and exponential generating functions.

Chapter III: Combinatorial parameters and multivariate generating functions.

Chapter IV: Complex analysis, rational and meromorphic asymptotics

Chapter VI: Singularity analysis of generating functions

Chapter VIII: Saddle-point asymptotics

Chapter IX: Multivariate asymptotics and limit laws

Piazza, Canvas, Gradescope

Outside discussions and logistics will mostly occur on piazza. Grades and a few basic things will be posted to the canvas page. Homework will be turned in via gradescope.

https://piazza.com/wustl/fall2023/math470/home

Exams

There will be two in-class midterm exams (September 27 and November 1) and a final exam. The final exam is Dec 15, 6-8pm.

Homework

There will be homework assignments every week or every other week done through gradescope:

www.gradescope.com

Homework solutions should be written up clearly and in detail. We may discuss possibilities for a project in place of a homework.

Collaboration: You may discuss the homework verbally with other students provided you have already given the homework a serious attempt. If you have already solved a problem and someone asks you about it, then any help you provide should consist of hints or suggestions and never complete solutions. In particular, homework should be written up independently and it should not be possible to tell who worked with whom. Do not search or post requests for solutions to HW. Do not post any course materials online. You may always post questions about homework via piazza discussions.

Dropping/Late policy: Your lowest homework score will be dropped and your second lowest score will be worth half the value of the other homework assignments. This policy is designed to take care of all instances where a student cannot complete an assignment on time so that the instructor does not need to make subjective judgement calls. Late homework will not be accepted so that solutions can be posted in a timely manner.

Grade breakdown

Homework: 60% Midterm exams: 20% Final exam: 20%

Letter grade breakdown: A+=(97,100], A=(93,97], A-=[90,93], similar for B,C,D. Finally,

F=[0,60).