Math 5021-Fall 2020

Complex Analysis I

$$\frac{1}{n!} = \frac{1}{2\pi i}\int_{r\partial\mathbb{D}} \frac{e^z}{z^{n+1}}\, dz \le \frac{e^n}{n^n}$$

General information

Location: Remote: https://wustl.zoom.us/j/97304254661
Time: MWF 2-2:50pm
Professor: Greg Knese
Office location: Remote/Zoom
Office hours: MWF 1-2pm, or zoom appointment
Email: geknese at wustl dot edu

Course description

An intensive course in complex analysis at the introductory graduate level. Math 5021 is a Ph.D. qualifying exam course in complex variables.
Prerequisite: Math 4111, 417 and 418, or permission of the instructor.

More details:

Math 5021 is an introduction to graduate complex analysis and serves as the basis for the Ph.D. qualifying exam in analysis. Topics covered include: A quick review of elementary properties of complex numbers, elementary complex functions (trigonometric functions, the logarithm, the exponential, argument, roots), analytic/holomorphic functions, complex differentiation, Cauchy-Riemann equations, complex line integrals, Cauchy's Theorem, Morera's Theorem, Goursat's Theorem, applications of Cauchy's Integral, Classification of singularities and zeros, meromorphic functions, zeros of holomorphic functions, holomorphic functions as geometric mappings, analytic functions and infinite series and products, Laurent Series, Residue Theorem, Real Integrals by Residues, conformal mappings, the Riemann Mapping Theorem, Compactness of families of analytic/meromorphic functions.

You should be proficient in undergraduate real analysis: naive set theory, epsilon-delta proofs, topology of R^n, topology of metric spaces, some general topology (e.g. connectedness), rigorous multivariable calculus (partial derivatives, inverse function theorem, Green's theorem). I will also assume that you are familiar with complex numbers and power series at the level of Calc II.

Textbook

There is no official textbook. I will provide notes and videos on canvas. The following standard complex analysis books will be posted on canvas.

Complex Analysis by Theodore Gamelin
Complex Analysis by Lars Ahlfors
Functions of one complex variables by John B. Conway
Complex Analysis by Stein and Shakarchi

Class Format

I will post videos and notes on canvas to deliver most of the content of the course. There may also be required or recommended reading. Everything course related will be available on course canvas page!

mycanvas.wustl.edu or https://wustl.instructure.com/

The zoom meeting link for the course is:

https://wustl.zoom.us/j/97304254661

During the scheduled class time I plan to have zoom meetings where I play the recorded videos (these will be snippets of length around 10 minutes). Then, there will be time to discuss and ask questions about the video. After that we will go through homework problems.

Although we may discuss problems it will still be up to you to write up thorough and complete solutions. The synchronous portion will be recorded and while it is technically optional it is important for people to attend and ask questions. The meetings are also a good way to force yourself to watch the videos and stay engaged.

Exams

The final exam is Jan 6, 2021, 3:30-5:30pm and will be conducted remotely. For math graduate students, this will constitute the qualifying exam in Complex Analysis, an important component of Ph.D. requirements.

There will be three remote non-cumulative midterms. (I think it will be important to get in the practice of taking remote exams so the final exam is not a surprise.) The dates will be October 9, November 9, December 16.

Exams will be open book/note but no dynamic assistance (people,computer programs) will be allowed.

Homework

There will be weekly homework assignments. These should be written up clearly and in detail preferably typed using LaTeX. You may discuss the homework verbally with other students provided you have already given the homework a serious attempt. If you have already solved a problem and someone asks you about it, then any help you provide should consist of hints or suggestions and not complete solutions.

In particular, homework should be written up independently and it should not be possible to tell who worked with whom. Do not search or post requests for solutions to HW.

Grade breakdown

Homework: 50%
Midterm exams: 30%
Final exam: 20%

Grade computations:

Your homework score will be computed as a weighted average. Suppose we have 10 homeworks. Let $a_0 \leq a_1 \leq \cdots \leq a_9$ be the increasing arrangement of your 10 scores. Then your homework score will be

$$\frac{a_1 + 2a_2 + \cdots 9a_9}{50}.$$

Notice that your lowest score a_0 is dropped and your highest score counts for $9/50 = 18\,\%$ of your grade. Your bottom 3 scores only count for 6% of your grade. For this reason there will be no “dropping” of scores except for the lowest score.
(If this grading scheme is too mathematical for you, you might not be in the right course!)

Your midterm score will be calculated in the same way but with three midterms. If the three scores are $a_0 \leq a_1 \leq a_2$, then your midterm score is

$$\frac{a_1 + 2a_2}{3}.$$

Notice your lowest score is dropped, your middle score counts for 1/3 and your highest score counts for 2/3.

Health Related information

If you become sick during the semester please let me know as soon as possible so we can make accommodations.

Course topics

The current plan is:

Complex numbers
Fractional Linear transformations
Special FLTs
Convergence, completeness, and power series
Holomorphic functions
Power series are holomorphic
The exponential function and friends
Cauchy integral formula for a circle
Holomorphic implies analytic
Cauchy-Riemann equations and harmonic functions
Inverse and implicit function theorem
Analyticity
Laurent series
Local behavior, zeros, poles, and essential singularities
Some global behavior
Curves, line integrals, forms, simple connectivity, winding numbers
Cauchy, Morera, Goursat theorems
Residue theorem
Real integrals from complex
Some asymptotics
Argument principle and applications
Harmonic functions again
Normal families
Riemann mapping theorem