Math 5022-Spring 2021
Complex Analysis II

$$\int_{\partial\mathbb{D}} \frac{dz}{z}$$

General information

Class zoom link: https://wustl.zoom.us/j/91306411146
Class zoom password: the value of the integral above
Time: TTh 1-2:15pm
Professor: Greg Knese
Office hour link: https://wustl.zoom.us/j/94477927372
Office hour password: same as class
Office hours: MW 4-5pm, F 3-4pm or zoom appointment
Email: geknese at wustl dot edu

Course description

Math 5022 is the continuation of graduate complex analysis and serves as part of the Ph.D. qualifying exam in analysis.
Prerequisite: Math 5021 or permission of the instructor.

More details:

Topics covered include: Elementary Riemann surfaces, Harmonic Functions, Harmonic Conjugates, Subharmonic Functions, Mean value property for harmonic functions and its converse, Poisson's formula, Dirichlet Problem, Approximation Theorems, Schwarz reflection, Infinite products (including Blaschke products), Weierstrass factorization, Mittag-Leffler, Jensen's formula, analytic continuation, monodromy, Special Functions (Γ, β, Riemann ζ, Elliptic), Prime Number Theorem, Stirling's formula, Basic properties of Several Complex Variables.

Additional topics that can be covered: Hardy Space Theory, Picard Theorems, Covering Spaces and Uniformization Theorem, Modular Functions

Textbook

There is no official textbook. I will provide notes and videos on canvas. The following standard complex analysis books will be posted on canvas.

Complex Analysis by Theodore Gamelin (I will mostly follow this book)
Complex Analysis by Lars Ahlfors
Functions of one complex variables by John B. Conway
Complex Analysis by Stein and Shakarchi

Class Format

Class will consist of remote and synchronous lectures via zoom. Lectures will be recorded and posted to canvas/zoom. There may also be required or recommended reading. Everything course related will be available on course canvas page!

mycanvas.wustl.edu or https://wustl.instructure.com/

The zoom meeting link for the course is:

https://wustl.zoom.us/j/91306411146

The password is the same as last semester. The office hour link is:

https://wustl.zoom.us/j/94477927372

Exams

The final exam is May 12, 2021, 1-3pm and will be conducted remotely. For math graduate students, this will constitute the qualifying exam in Complex Analysis II, an important component of Ph.D. requirements.

There will be three take-home non-cumulative exams. You will have approximately 2 days for each of these.

Exams will be open book/note but no dynamic assistance (people,computer programs) will be allowed.

Homework

There will be weekly homework assignments. These should be written up clearly and in detail preferably typed using LaTeX. You may discuss the homework verbally with other students provided you have already given the homework a serious attempt. If you have already solved a problem and someone asks you about it, then any help you provide should consist of hints or suggestions and not complete solutions.

In particular, homework should be written up independently and it should not be possible to tell who worked with whom. Do not search or post requests for solutions to HW.

Grade breakdown

Homework: 60%
Midterm exams: 20%
Final exam: 20%

Grade computations:

Your homework score will be computed as a weighted average. Suppose we have 10 homeworks. Let $a_0 \leq a_1 \leq \cdots \leq a_9$ be the increasing arrangement of your 10 scores. Then your homework score will be

$$\frac{a_1 + 2a_2 + \cdots 9a_9}{45}.$$

Notice that your lowest score a_0 is dropped and your highest score counts for $9/45 = 20\,\%$ of your grade. Your bottom 3 scores only count for 6.7% of your grade. For this reason there will be no “dropping” of scores except for the lowest score.
(If this grading scheme is too mathematical for you, you might not be in the right course!)

Your lowest midterm exam score will be dropped. (Note this is different from last semester.)

Health Related information

If you become sick during the semester please let me know as soon as possible so we can make accommodations.

Course topics

Harmonic functions
Schwarz reflection principle
Subharmonic functions
Dirichlet problem
Analytic continuation
Introduction to Riemann surfaces
Normal families
Runge’s theorem
Mittag-Leffler theorem
Blaschke products
Weierstrass product theorem
Laplace transforms and Stirling’s formula
Dirichlet series
Prime number theorem
Uniformization theorem

Lots of options after that:
Jordan curve theorem and Caratheodory’s theorem
Hardy spaces
Puiseux’s theorem
Meromorphic functions on Riemann surfaces