Math 523-Spring 2023

Topics in Analysis: Dirichlet Series

$$\zeta(s) = \sum_{n \geq 1} n^{-s} = \prod_{p} \left(1 - p^{-s}\right)^{-1}$$

General information

Class location: Life Sciences 118
Class time: MWF 12 - 12:50pm
Professor: Greg Knese
Office location: Cupples I room 214
Office hours: TBA
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Course description

We will begin with some historical gems in analytic number theory such as Dirichlet's theorem on primes in arithmetic progressions and the prime number theorem. We will eventually move on to study some purely analytic problems related to Dirichlet series. Highlights may include:

—Bohr's problem on the abscissa gap, the Bohnenblust-Hille theorem and inequalities, random polynomials, the Bohr radius problem.

—Asymptotics of Mellin transforms and connections to analytic combinatorics

—Beurling's dilation completeness problem and the work of Hedenmalm-Lindqvist-Seip

—Bohr's theory of almost periodic functions

—Voronin and Bagchi theorems on universality of the zeta function.

—Additional topics in analytic number theory.

Prerequisites: graduate complex analysis, graduate measure theory and functional analysis, undergraduate algebra, or permission of the instructor.

Books

I will follow a variety of books. Here are some:

—Serre's A course in arithmetic (for Dirichlet's theorem)
—Dirichlet series and holomorphic functions in high dimensions by Defant et al.
—Diophantine approximation and Dirichlet series by Queffelec and Queffelec
—Dirichlet series — Notes by Prof. John E. McCarthy

—The theory of the Riemann zeta function by Titchmarsh
—Excursions in multiplicative number theory by Ramaré
—Multiplicative number theory by Davenport
—The Distribution of prime numbers by Koukoulopoulos
—Introduction to Analytic and Probabilistic Number theory by Tenenbaum

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Grades

Grades will be based on attendance, participation, and on a presentation at the end of the course. Presentation topics will be discussed throughout the course. From time-to-time we may have days where we work on problems in class.